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Correcting a Significant and Consistent Error in the Modal Damping Obtained using Transient Vibration Data

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ABSTRACT

This report investigates the experimental vibration technique of impact excitation when used to obtain frequency response functions. It is shown that current practices introduce a consistent error in the derived modal damping estimates. The error can be significant, with levels of damping being wrongly predicted by a factor of three or more. The report identifies the source of the error. It then derives and presents a simple correction to be applied to the observed modal damping estimates. The procedure is demonstrated by experiment.

INTRODUCTION

Traditionally it has been assumed that experimental frequency response functions are properties of the structure under test, and are independent of the test method used to measure them. In this way, the modal parameters obtained from an experimental modal analysis should be independent of the vibration test method. Variations between results from different tests are typically assigned to "experimental error," including signal noise as well as such variables as experimental technique and boundary conditions.

For many structures, the natural frequencies and mode shapes can usually be obtained with high confidence. Observation of the relative mode shapes of a structure can also give further confidence in the experimental and analysis procedures. "Smooth" and continuous mode shapes are good indicators that the data have been measured with a repeatable technique between test points. On the other hand, it is common to see variations in modal damping between different tests. This variation is often attributed to the fact that damping is a difficult quantity to measure. Modal analysis assumes a simple form of damping (usually viscous or hysteretic) whereas the actual energy dissipation mechanism is generally more complicated, often nonlinear, and may vary between different experiments.

There is, however, a consistent error introduced into the data obtained using transient excitation methods. Because the error is consistent, its effect cannot be seen by looking solely at the modal data. For example, damping levels may seem realistic. But the error changes the modal damping obtained from the experimental data. This consistent error can be 'backed-out' of the resulting parameters, leading to much improved damping estimates.

TEST PROCEDURE SUMMARY

The consistent error is introduced at the data capture stage, and is a direct result of the different nature of the impulsive force, and exponential response signals. A brief summary of the relevant aspects of a transient test procedure is therefore in order.

Typically, a reference accelerometer measures the response resulting from a short duration impact excitation. Both the force and acceleration signals are conditioned, then digitized by a spectrum analyzer. At this stage, the time domain signals can be inspected. The force signal will be a sharp pulse near the start of the time record. After the pulse is complete, the force signal is essentially zero, although there will be some signal and digitization noise. The acceleration response is quiescent up to the time when the hammer impacts the structure. During the short time coincident with the hammer impact, the acceleration levels rapidly increase. Thereafter, the acceleration signal is a multi-modal impulse response function, with an overall negative exponential envelope.

WINDOWING

Before these time signals are Fourier transformed by the analyzer, windows are applied to reduce the leakage caused by only having a finite-length time record. Both signals are basically zero at the start of the time record, and therefore the primary purpose of the applied windows is to attenuate the signals at the end of the time record. Since the force signal is already zero at the end of the time record, no window is specifically needed, although it is common practice to use either a square or exponential window. Conversely, the acceleration response signal may have significant amplitude at the end of the time record, and typically an exponential window is used to attenuate the signal. An exponential window is chosen because it matches the overall shape of the acceleration signal. Also it provides minimal attenuation at the start of the time record, where much of the important information is concentrated. Most analyzers permit the time constant for the exponential window to be varied because the acceleration for a lightly damped structure needs to be attenuated more than that for a heavily damped structure. Similarly, when measuring a high frequency bandwidth the analyzer only captures a short time record, and the signal requires more window attenuation than measurements from the same structure, but to a lower frequency (longer time). The exponential window has the form:

$$\mathrm{w}(t) = e^{-t/\tau} \tag{1}$$

where t is the time and τ is a time constant. We investigate the effect of the window by looking at the response of a viscously damped single degree of freedom system. The acceleration impulse response can be shown in the form:

$$a(t) = A\,e^{-\zeta_r \omega_r t} \sin(\omega_r \sqrt{1-\zeta_r^2}\,t + \phi) \tag{2}$$

where ω_r is the circular natural frequency and ζ_r is the viscous damping ratio. When the measured acceleration signal is multiplied by the window function, the resulting modified acceleration time signal, a*, is given by:

$$a^*(t) = a(t).w(t) = A\,e^{-(\zeta_r \omega_r + 1/\tau)t} \sin(\omega_r \sqrt{1-\zeta_r^2}\,t + \phi) \tag{3}$$

We therefore see that the effect of the exponential window is to increase the rate at which the acceleration signal decays. This represents an increase in the amount of damping apparent in the system. The force signal does not show the same increase in damping; a signal that is predominantly zero can be multiplied by any window with almost no change. Currently digital spectrum analyzers do not remove the added damping from the measured frequency response functions. It is therefore necessary to determine the effect the change to the acceleration signal causes on the modal data, and to identify how the effect can be removed.

MODAL ANALYSIS

Modal damping is an indication of the energy dissipation potential of a particular mode. The level of modal damping is based on the lossiness of the structural material and boundaries, as well as the distribution of strain throughout the structure. This is the 'forward' problem - a theoretical determination of modal damping based on mode shapes. An experimental modal analysis is the 'reverse' problem. When modal parameters are estimated from experimental data, energy dissipation is one of the first properties to be determined. The mode shapes (or, more strictly, the modal constants) are then determined from the complex magnitudes of the frequency response functions *and the*

previously determined levels of damping. This procedure can be demonstrated by considering the Nyquist plot for a structure near resonance. For theoretical consistency with the assumed viscously damped model, we consider here the mobility frequency response function.

It is well known that when the mobility near resonance is plotted in the complex plane, the result is a circle of diameter $A_{rij} / 2\zeta_r\omega_r$, where A_{rij} is the modal constant for the r-th natural frequency, and between the i-th and j-th spatial coordinates, ζ_r is the modal viscous damping ratio, and ω_r is the circular natural frequency. When using the simplest form of circle-fit modal analysis, the following procedure is adopted:

a. The natural frequency ω_r is located as the frequency at which the rate of change of phase (with respect to the center of the circle) is a maximum.

b. The viscous damping ratio is estimated from:

$$\zeta_r = \frac{(\omega_2^2 - \omega_1^2)}{2\,\omega_r\,\{\tan(\theta_2/2) + \tan(\theta_1/2)\}} \tag{4}$$

where θ_1 is the phase angle (measured at the center of the circle) between the frequency response function measurements at ω_1 and ω_r, with $(\omega_1 < \omega_r)$. Similarly for θ_2 with $(\omega_2 > \omega_r)$.

c. The modal constant is calculated from the diameter of the circle:

$$A_{rjk} = (\text{circle diameter}) \, . \, 2\,\zeta_r\,\omega_r \tag{5}$$

It is important to note the difference between the 'forward' theoretical approach, and the 'reverse' experimental method. The 'forward' problem determines the modal constant independently from the damping, and the damping is based on the mode shape.

Conversely, for the experimental modal analysis, the modal constant depends on an estimated modal damping.

CORRECTING FOR THE WINDOW EFFECTS

We therefore see that the consistent increase in damping caused by the acceleration window function not only changes the modal damping, but potentially also changes the modal constants. We therefore need to determine the effect of the window, and how to remove it. Ideally, the effect would be removed as a broadband change to the measured frequency response functions, prior to modal analysis of the data. The simple multiplication of the acceleration signal and the window function in the time domain represents a convolution in the frequency domain. The most common method used to deconvolve two signals in the frequency domain is to Fourier transform to the time domain, perform a division, and then transform the result back to the frequency domain. Clearly this deconvolution procedure is unsuitable for the acceleration signal - the net effect is that the acceleration signal is not windowed, and leakage is restored. Similarly, more complex frequency-based deconvolution methods would also reintroduce leakage. We therefore see that broadband elimination of the window effect is impractical. The alternative is to consider narrowband corrections.

Comparing Eqns. (2) and (3) we see that the exponential envelope for the impulse response function of a single degree of freedom system depends on the circular natural frequency and modal damping. For the windowed data, the rate also depends on the window time constant, τ. The modal damping estimated from the measured data, ζ_M is therefore related to the actual modal damping, ζ_r by:

$$\omega_r \zeta_M = \zeta_r \omega_r + 1/\tau \quad \textbf{(6)}$$

The corrected modal damping can therefore be determined from the measured damping:

$$\zeta_r = \zeta_M - \frac{1}{2\pi f_r \tau} \quad \textbf{(7)}$$

where f_r is the natural frequency in Hz. The applied correction can be significant at low frequencies, or when a short time constant exponential window is used.

We now turn our attention to the modal constants. At first sight, Eqn. (5) seems to indicate that the measured modal constant depends on the measured damping ratio. That observation is partially correct, in that if there is some error in the measurement of damping, this will reflect in a consistent error in the modal constant. However, we also have to note that both the frequency response function data and the measured damping are modified by the exponential window. Specifically, the diameter of the Nyquist circle is inversely proportional to the viscous damping ratio. The diameter of the Nyquist circle for the data obtained with an exponential window, D_M, can be related to the diameter that would have been obtained without using a window, D_O:

$$D_M = D_O \frac{\zeta_r}{\zeta_M} \quad \textbf{(8)}$$

The measured modal constant, $(A_{rij})_M$ is determined from the Eqn. (5), the diameter of the windowed data, and the measured (incorrect) damping ratio:

$$(A_{rjk})_M = D_M \,.\, 2\zeta_M \omega_r \quad \textbf{(9)}$$

Substituting Eqn. (8) and comparing with Eqn. (5):

$$(A_{rjk})_M = D_O \frac{\zeta_r}{\zeta_M} \,.\, 2\zeta_M \omega_r = D_O \,.\, 2\zeta_r \omega_r = A_{rjk} \quad \textbf{(10)}$$

We therefore see that the modal constants obtained from the windowed acceleration data are the same as the correct modal constants. The consistent error introduced into the

acceleration data and measured damping cancels out in the calculation of the modal constant.

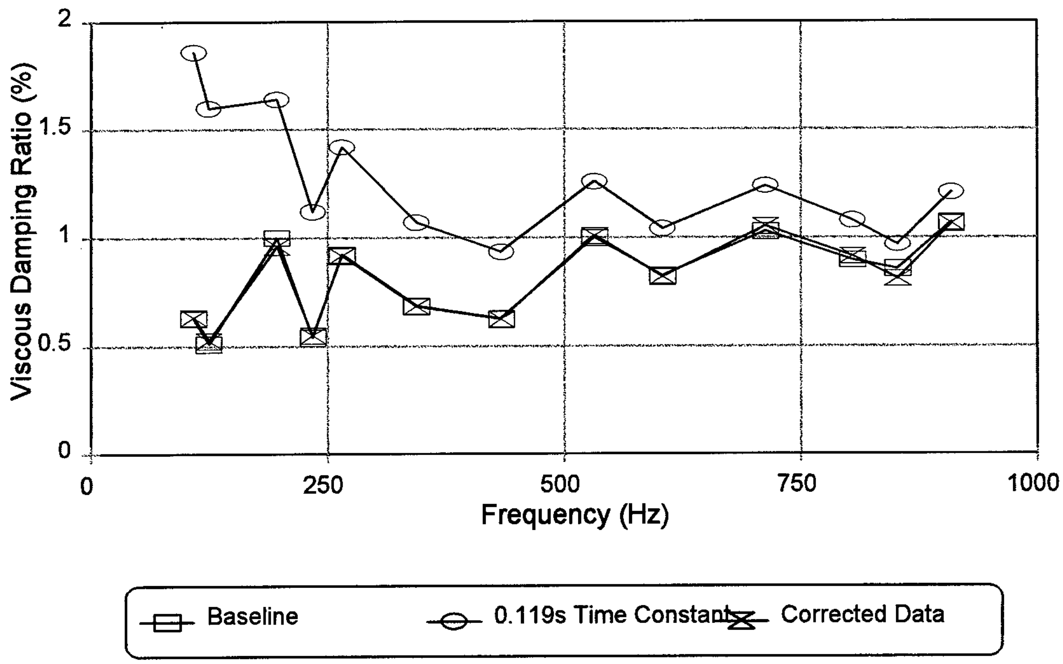
DAMPING EXAMPLE

A steel plate with a viscoelastic damping material on one side was tested using transient methods. For the first test, the exponential window time constant was set at 110 seconds. This large constant made the window ineffective, and meant the corrections of Eqn. (7) were very small. This data set thus yielded very close approximations to the correct damping values, and is referred to as the 'baseline' data set. The modal damping values were estimated using a commercial modal analysis package.

For the second test, an exponential window time of 0.119 seconds was used, this being more representative of values typically used for resonant structures. The modal damping values were again estimated using a commercial modal analysis package, and the correction of Eqn. (7) was applied.

The estimated viscous damping ratios are compared in the table, and graphically in the figure. There is a significant difference between the baseline modal damping values and the uncorrected values obtained using an exponential window. When the correction of Eqn. (7) is applied, there is almost no difference between the two sets of modal damping ratios.

Natural Frequency (Hz)	Viscous Damping Ratio (%)			Ratio of damping to baseline	
	Baseline	Windowed, uncorrected	Windowed, corrected	Windowed, uncorrected	Windowed, corrected
108.9	0.63	1.86	0.63	2.95	1.00
124.3	0.51	1.60	0.52	3.14	1.03
196.3	1.00	1.64	0.96	1.64	0.96
234.6	0.54	1.12	0.55	2.07	1.02
265.6	0.92	1.42	0.92	1.54	1.00
344.1	0.69	1.07	0.68	1.55	0.99
433.2	0.63	0.93	0.62	1.48	0.99
532.3	1.00	1.26	1.01	1.26	1.01
604.1	0.83	1.04	0.82	1.25	0.99
713.1	1.03	1.24	1.05	1.20	1.02
804.8	0.90	1.08	0.91	1.20	1.02
852.8	0.86	0.97	0.81	1.13	0.95
909.5	1.07	1.21	1.06	1.13	0.99



CONCLUSIONS

Standard transient vibration test methods necessitate windowing of time domain signals prior to Fourier transforming. typically, an exponential window is used for the response signal. The differences in the physical nature of the excitation signal (short duration) and response signal (long duration) means the effect of the window is significantly different on each channel. The net effect of the window is to over-estimate the amount of modal damping. Natural frequencies and modal constants are not effected.

The effect of the window is broadband, but making a broadband correction to the measured frequency response functions is not trivial. However, the modal damping represents energy dissipation in a narrow frequency band. This report determined a simple correction to be applied to modal viscous damping ratios.

REFERENCES

Ewins, D.J., "Modal Testing: Theory and Practice," Research Studies Press Ltd., Letchworth, Hertfordshire, England, 1986.

> All of these [impulsive/transient] latter possibilities offer shorter testing times but great care must be exercised in their use as there are many steps at which errors may be incurred by incorrect application.
>
> 3.3.6 Hammer or Impactor Excitation
>
> Another popular method of excitation is through use of an impactor or hammer. Although this type of test places greater demands on the analysis phase of the measurement process, it is a relatively simple means of exciting the structure into vibration.
>
> [actually talks about hammer mass/tip, position of hits, orientation, multiple hits "bounce", overload, elastic range]

Halvorsen, W.G. and Brown, D.L., "Impulsive Technique for Structural Frequency Response Testing," JSV 8-21 Nov 1977

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